Problem:

Solve the Cauchy problem, using the operational method:

$y'' - 5y' + 6y = 6t - 5, \qquad y(0) = 1, \qquad y'(0) = 3.$

Solution

Applying the Laplace transform to both parts of the equation, we pass to Laplace transforms: let $y(t) \risingdotseq Y(p)$ $\Rightarrow$ applying the theorem of differentiation of the signal $\Rightarrow$ $y'(t) \risingdotseq pY(p) - y(0) = pY(p) - 1$, $y''(t) \risingdotseq$ $\risingdotseq p^2Y(p) - py(0) - y'(0) = p^2Y(p) - p - 3$, from the Laplace Transform Table we see:

$t \risingdotseq \frac{1}{p^2}, 1 \risingdotseq \frac{1}{p} \Rightarrow$ we obtain: $p^2Y(p) - p - 3 - 5pY(p) + 5 + 6Y(p) = \frac{6}{p^2} - \frac{5}{p} \Rightarrow$

$$\Rightarrow Y(p) = \frac{1}{p^2 - 5p + 6}\left(p - 2 + \frac{6}{p^2} - \frac{5}{p}\right) = \frac{p^3 - 2p^2 - 5p + 6}{p^2(p-2)(p-3)} = \frac{p^3 - 3p^2 + p^2 - 5p + 6}{p^2(p-2)(p-3)} =$$

$$= \frac{p^2(p-3)}{p^2(p-3)(p-2)} + \frac{1}{p^2} = \frac{1}{p-2} + \frac{1}{p^2} \Rightarrow \quad (*)\ Y(p) = \frac{1}{p-2} + \frac{1}{p^2},$$

from the Laplace Transform Table we see:

$e^{2t} \risingdotseq \frac{1}{p-2}$, $t \risingdotseq \frac{1}{p^2} \Rightarrow$ in $(*)$ passing to the signals, we obtain the desired solution of the initial problem:

$\boxed{y(t) = e^{2t} + t}$.