Problem:

Determine the character of the resting point of the following system:

$x' = x + 2y, y' = -3x + y.$

Solution:

$$\begin{cases} x' = x + 2y & M(x, y) = x + y = 0 \\ y' = -3x + y & N(x, y) = -3x + y = 0 \end{cases} \Rightarrow \text{ the resting point } x = y = 0, M(0;0) = N(0;0) = 0.$$

Let's find the eigenvalues of the matrix of the system. $A = \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}$, $\det(A - \lambda I) = \left| \begin{pmatrix} 1-\lambda & 2 \\ -3 & 1-\lambda \end{pmatrix} \right| = 0$, $(1-\lambda)(1-\lambda) + 6 = 0, (\lambda - 1)^2 = -6, \lambda = 1 \pm i\sqrt{6}$. Since $Re(\lambda_1) = Re(\lambda_2) = 1$, then the singular point will be the focus, and integral curves have the form of a spiral, twisting around the beginning of the coordinates. Next $Re(\lambda_1) = Re(\lambda_2) = 1 > 0 \Rightarrow$ according to Lyaponv's first theorem the resting point of the system $(*)$ is unstable.

Answer: the resting point of the system is unstable.