



Problem:

The random variable  $X$  is uniformly distributed on the segment  $[-4, 2]$ . Find  $E(e^{4X})$ .

Solution:

$X$  is uniformly distributed on  $[-4, 2] \Rightarrow$  the probability density will be

$$f(x) = \begin{cases} \frac{1}{2 - (-4)}, & -4 \leq x \leq 2 \\ 0, & x \notin [-4; 2] \end{cases} = \begin{cases} \frac{1}{6}, & x \in [-4; 2] \\ 0, & x \notin [-4; 2] \end{cases}, \quad Y = e^{4X} = g(X), \quad \text{where } g(t) = e^{4t}$$

$\Rightarrow$  for the expected value of  $Y = g(X)$  we have the formula

$$E(e^{4X}) = \int_{-\infty}^{+\infty} g(x) \cdot f(x) dx = \int_{-4}^2 e^{4x} \cdot \frac{1}{6} dx = \frac{e^{4x}}{4 \cdot 6} \Big|_{-4}^2 = \frac{1}{24} (e^8 - e^{-16}) \Rightarrow \boxed{E(e^{4X}) = \frac{e^8 - e^{-16}}{24}}$$