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Problem:

It's known that for independent random variables $X_1, \cdots, X_4$ their expected values are

$E(X_i) = -2$, the dispersions are $D(X_i) = 1, i = 1, \cdots, 4$. Find the dispersion of the product $D(X_1; \cdots X_4)$.

Solution:

For the dispersion of two independent random variables $X, Y$ we have the formula:

$D(X \cdot Y) = D(X) \cdot D(Y) + E^2(X) \cdot D(Y) + E^2(Y) \cdot D(X)$, $X_1, \cdots, X_4$ are independent $\Rightarrow X_1 \cdot \ldots \cdot X_3, X_4$ are also independent $\Rightarrow$ $D(X_1 \cdot \cdots \cdot X_4) = D(X_1 \cdot \cdots \cdot X_3) \cdot D(X_4) + E^2(X_1 \cdots X_3) \cdot D(X_4) + E^2(X_4) \cdot D(X_1 \cdots X_3) =$ $\boxed{E(X_1 \cdot X_2 \cdot X_3) = E(X_1)E(X_2)E(X_3) = -8 \Rightarrow} = 5 \cdot D(X_1 \cdots X_3) + (-8)^2 \cdot 1 = 5D(X_1 \cdots X_3) + 64$, similarly, $D(X_1 \cdot X_2 \cdot X_3) = D(X_1 \cdot X_2) \cdot D(X_3) + E^2(X_1 \cdot X_2) \cdot D(X_3) + E^2(X_3) \cdot D(X_1 \cdot X_2) = 5D(X_1 \cdot X_2) + 4^2$, $D(X_1 \cdot X_2) = D(X_1) \cdot D(X_2) + E^2(X_1) \cdot D(X_2) + E^2(X_2) \cdot D(X_1) = 1 + 4 + 4 = 9 \Rightarrow$

$\Rightarrow D(X_1 \cdot X_2 \cdot X_3) = 5 \cdot 9 + 16 = 61 \Rightarrow D(X_1 \cdots X_4) = 5 \cdot 61 + 64 = 369.$

Answer: 369.