Problem:

Let the random variable ξ be the sum of the amount of the loan, requested by the bank's clients per day. This amount can change in the range from 0 to 1000000 (rubles). The bank immediately withdraws the commissions of 1%. What function will describe the amount of commissions, received by the bank, depending on the requested loan amount (the random variable τ)? Find the distribution function of the random variable $F_\tau(z)$.

Solution:

ξ is equally distributed on $[0;\ 1000000] \Rightarrow$

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{x}{10^6}, & 0 < x \le 10^6 \\ 1, & x > 10^6 \end{cases}, \qquad f_\xi(x) = \begin{cases} \dfrac{1}{10^6}, & x \in (0; 10^6] \\ 0, & x \notin (0; 10^6] \end{cases}$$

The amount of the commissions will be $\tau = 0.01\xi$, the distribution function of τ will be:

$$F_\tau(x) = p(\tau < x) = p(0.01\xi < x) = p(\xi < 100x) = F_\xi(100x) = \begin{cases} 0, & 100x \le 0 \\ \dfrac{100x}{10^6}, & 0 < 100x \le 10^6 \\ 1, & 100x > 10^6 \end{cases} \Rightarrow$$

$$F_\tau(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{x}{10^4}, & 0 < x \le 10^4 \\ 1, & x > 10^4 \end{cases}$$